We note that analogous results also occurred for other model thicknesses and values of the parameters of the oncoming stream. Here the maximum value of the derivative $dT_W/d\tau$ reached in the numerical experiments was 10^5 °K/sec.

NOTATION

t, τ, time; x, y, coordinates; η, ξ, dimensionless coordinates; u, v, components of the velocity vector; ρ, density; P, pressure; C_k, concentration by weight of k-th component; D_{12}, coefficient of binary diffusion; T, temperature; λ, coefficient of thermal conductivity; M, molecular weight; q, heat flux; ε, emissivity; h, h_1, integration step along the spatial coordinate; Δt, Δτ, integration step in time; δ, thickness of solid body; μ, viscosity; I, total enthalpy; R_B, blunting radius of solid body; s, r, bunching parameters of difference grid; e, index of external limit of boundary layer; W, index of surface over which flow occurs.

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STEFAN-TYPE PROBLEM FOR SUBLIMATION IN A POROUS BODY

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UDC 536.423.16

The mathematical formulation of the problem of heat and mass transfer during evaporation from a semiinfinite porous body consisting of parallel capillaries is given. The asymptotic solution of the problem is obtained for large and small time periods.

It was shown earlier [1] that the velocity of passage of the evaporation front v from capillaries (in the case of free-molecular regime of vapor flow) depends substantially on the depth of the evaporation zone in the porous body:

$$v = \frac{d\xi}{dt} = \frac{v_0}{1 + \xi/2r}, \tag{1}$$

where [2]

$$v_0 \approx a \exp\{-LA/RT\}. \tag{2}$$

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 6, pp. 1015-1018, December, 1977. Original article submitted April 5, 1977.

The formulation of the Stefan-type heat problem describing the process of heat and mass transfer during evaporation (sublimation) from a model porous body, consisting of parallel capillaries filled with evaporating material, in which free-molecular regime of vapor flow occurs, is natural.

Let us consider a semiinfinite porous body ($x > 0$) heated by a constant heat flux q_0. The mathematical formulation of the Stefan-type problem mentioned above has the form

$$\frac{\partial T_1}{\partial t} = a_1 \frac{\partial^2 T_1}{\partial x^2}, \quad 0 < x < \xi, \tag{3}$$

$$\frac{\partial T_2}{\partial t} = a_2 \frac{\partial^2 T_2}{\partial x^2}, \quad \xi < x < \infty. \tag{4}$$

$$-\lambda_1 \frac{\partial T_1}{\partial x}\Big|_{x=0} = q_0, \tag{5}$$

$$-\lambda_1 \frac{\partial T_1}{\partial x}\Big|_{x=\xi} + \lambda_2 \frac{\partial T_2}{\partial x}\Big|_{x=\xi} = \Pi \rho_2 L \frac{d\xi}{dt}, \tag{6}$$

$$\frac{d\xi}{dt} = \frac{a \exp\{-LA/RT_*\}}{1 + \xi/2r}. \tag{7}$$

$$\frac{\partial T_2}{\partial x}\Big|_{\to\infty} = 0, \tag{8}$$

$$T(x, 0) = T_0. \tag{9}$$

It is easy to note that the problem (3)-(9) essentially differs from the problem of sublimation of a semiinfinite solid [2] by the presence of two zones and expression (7) for v, as well as from the two-phase problems of melting (solidification) of a body by the absence of a constant temperature of the phase transition at the interface of the phases $x = \xi$. Furthermore, the velocity v at the initial time ($t = 0$) is finite, since (7) takes into consideration the kinetics of mass transfer during evaporation from the capillary. In this sense there is an analogy with the crystallization problem described in [3].

For obtaining the asymptotic expression for v and $T_* = T|_{x=\xi}$ for large values of t we shall make use of the integral methods of solution of heat and mass transfer problems in the presence of phase transformations [4]. Since the velocity $d\xi/dt$ is small in this problem, we shall assume that the temperature distribution differs little from the corresponding distribution for $d\xi/dt = 0$. Therefore, after integration of the left- and right-hand sides of Eqs. (3) and (4) over x and using boundary conditions (5), (6), (8), in the remaining integral terms we substitute the approximate expressions

$$T_1 = T_* + \frac{\xi q_0}{\lambda_1}\left(1 - \frac{x}{\xi}\right), \quad T_2 = (T_* - T_0)\,\Phi^*\left(\frac{x - \xi}{\sqrt{2a_2 t}}\right) + T_0.$$

As a result, we obtain the differential equation

$$\xi \frac{d\xi}{dt}\frac{q_0}{a_1} + \frac{d\xi}{dt}[\Pi\rho_2 L + c_2\rho_2 (T_* - T_0)] + \xi \frac{dT_*}{dt} c_1\rho_1 + 2c_2\rho_2 \sqrt{\frac{a_2}{\pi}}\,\frac{d[(T_* - T_0)\sqrt{t}]}{dt} = q_0. \tag{10}$$

Equations (10) and (7) permit a determination the desired quantities $\xi(t)$ and $T_*(t)$.

If it is assumed that for large times T_* changes with time slower than ξ, then from (10) we easily obtain the equation

$$\frac{q_0}{2a_1}\xi^2 + \Pi\rho_2 L\xi + c_2\rho_2\xi(T_* - T_0) + 2c_2\rho_2\sqrt{\frac{a_2}{\pi}}(T_* - T_0)\sqrt{t} = q_0 t. \tag{11}$$

Furthermore, under the same assumptions we can approximately integrate (7):

$$\xi = 2r\left(\sqrt{1 + t\frac{a\exp\{-LA/RT_*\}}{r}} - 1\right) \approx 2r\sqrt{t\frac{a}{r}\exp\{-LA/RT_*\}}. \tag{12}$$

Substituting (12) into (11) we obtain

$$q_0 - 2\frac{q_0}{a_1} ra \exp\{-LA/RT_*\} =$$

$$= 2\frac{[\Pi\rho_2 L + c_2\rho_2(T_* - T_0)]\sqrt{ra\exp\{-LA/RT_*\}} + c_2\rho_2\sqrt{\frac{a_2}{\pi}}(T_* - T_0)}{\sqrt{t}}. \tag{13}$$

At large times the method of successive approximations can be used for solving (13). In the zero-order approximation we have

$$1 - \frac{2}{a_1} ra \exp\{-LA/RT_*\} = 0$$

or

$$T_* = \frac{LA}{R}\frac{1}{\ln\left(\frac{2ra}{a_1}\right)}, \tag{14}$$

$$\frac{d\xi}{dt} = \sqrt{\frac{a_1}{2t}}. \tag{15}$$

We note that the asymptotic value of the temperature T_* (14) depends on the thermal diffusivity of only the dry zone, since for sufficiently large t the larger part of the heat goes into heating this zone.

The first approximation gives

$$T_* = \frac{LA}{R}\frac{1}{\ln\left[1\Big/\left(\frac{a_1}{2ra} - \frac{B}{\sqrt{t}}\right)\right]}, \tag{16}$$

$$\frac{d\xi}{dt} = \sqrt{\frac{a_1}{2t} - \frac{Bar}{t^{3/2}}},$$

$$B = \frac{a_1}{q_0 ra}\left\{\left[\Pi\rho_2 L + c_2\rho_2\left(\frac{LA}{R}\frac{1}{\ln(2ra/a_1)} - T_0\right)\right]\sqrt{\frac{a_1}{2}} + \right.$$

$$\left. + c_2\rho_2\sqrt{\frac{a_2}{\pi}}\left(\frac{LA}{R}\frac{1}{\ln(2ra/a_1)} - T_0\right)\right\}. \tag{17}$$

For small values of t the thickness of the dry zone is smaller than the diameter of the capillary and the heat spent in heating the layer is negligibly small; we can make use of the solution of [3] for the problem of sublimation from the surface of a semiinfinite solid body:

$$\frac{d\xi}{dt} = v_n + \frac{1}{\sqrt{a_2}}\frac{d\xi_1}{dt} + \ldots, \tag{18}$$

$$\frac{LA}{RT_*} = \ln\frac{a}{v_n} - \frac{1}{\sqrt{a_2}}\frac{v_1}{v_n}, \tag{19}$$

where

$$v_1(t) = \frac{v_n \ln^2(a/v_n)}{LA/R}\left(\frac{q_0}{c_2\rho_2} - \frac{L}{c_2}v_n\right)\frac{2}{\sqrt{\pi}}\sqrt{t},$$

$$\xi_1(t) = \frac{v_n \ln^2(a/v_n)}{LA/R}\left(\frac{q_0}{c_2\rho_2} - \frac{L}{c_2}v_n\right)\frac{4}{3\sqrt{\pi}}t^{3/2}.$$

Thus it follows from expressions (14)-(19) that the velocity of the transition front first increases with time and then after a certain t reaches its maximum value, and finally at sufficiently large times it decreases proportional to $1/\sqrt{t}$. The temperature at the evaporation front T_* increases with time and at large values of t it tends to a certain constant value (14). However, actually this asymptotic value of T_* is not attained after a certain finite time interval during which the temperature of the surface of the body does not exceed the melting temperature of the basic material of the porous body; i.e., T_* is the function of time.

NOTATION

r, the radius of the capillary; L, evaporation heat; A, atomic weight; T, temperature; R, gas constant; ξ, coordinate of the evaporation front; a, speed of sound; Π, porosity; λ, thermal conductivity; $\Phi^*(y) = (2/\sqrt{\pi}) \cdot \int_g^\infty \exp(-z^2)dz$; $v_n = v(0)$. The indices 1 and 2 pertain, respectively, to the parameters of the dry zone and of the initial body.

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APPLICATION OF THE TIKHONOV METHOD TO SOLVE THE INVERSE HEAT-CONDUCTION PROBLEM FOR A MELTING PLATE WITH MELT ENTRAINMENT

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UDC 526.24.02

An algorithm is proposed for the solution of the inverse heat-conduction problem for a melting plate with instantaneous removal of the liquid phase.

The inverse heat-conduction problem for a domain (plate) with moving boundary reduces to finding the law of motion of the melting solid body and two heat fluxes on the plate boundaries on the basis of the known temperature change in two interior points. The method of solving such a problem, proposed in one of the author's papers [1], is extended in this paper to the case when the method of successive intervals is inapplicable, i.e., when either the depth of the points x_1 and x_2 does not correspond to the magnitude of the time interval during which the temperatures are measured ($x_1^2/2l^2 > a\Delta\tau/l^2$ or $(l - x_2)^2/2l^2 > a\Delta\tau/l^2$) or the measurement errors are large $|\hat{t}(x_1, \tau) - t(x_1, \tau)|$. Underlying the method proposed is the more general approach to the solution of incorrect problems of mathematical physics proposed by Academician A. N. Tikhonov [2, 3].

Let us consider the temperature field in a plate heated by a heat flux of density $q_1(\tau)$, where the flux density $q_2(\tau)$ emerges through the opposite face of the plate.

Prior to the beginning of melting ($\tau < \tau_m$) this temperature field is subject to the heat-conduction equation

$$\lambda \frac{\partial^2 t}{\partial x^2} = c\rho \frac{\partial t}{\partial \tau}, \quad 0 \leqslant x \leqslant l, \quad 0 < \tau < \tau_m \tag{1}$$

with the boundary conditions

$$-\lambda \frac{\partial t}{\partial x}\bigg|_{x=0} = q_1(\tau), \tag{2}$$

$$-\lambda \frac{\partial t}{\partial x}\bigg|_{x=l} = -q_2(\tau) \tag{3}$$

and the initial condition

$$t(x, 0) = \varphi(x). \tag{4}$$

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